An entropy-based method for resource leveling

Symeon Christodoulou  
*University of Cyprus, Department of Civil and Environmental Engineering, Nicosia, Cyprus*

Georgios Ellinas  
*University of Cyprus, Department of Electrical and Computer Engineering, Nicosia, Cyprus*

Anastasia Michaelidou-Kamenou  
*University of Cyprus, Department of Civil and Environmental Engineering, Nicosia, Cyprus*

**Keywords:** resource-levelling, entropy

The paper revisits the Minimum Moment Method – a heuristic commonly used in resource levelling – and restates it as an Entropy Maximization problem. The proposed entropy-maximization method makes use of the general theory of entropy and two of its principal properties (subadditivity and maximality) to redefine resource levelling in terms of the entropy caused, and to develop an algorithmic framework for its solution. The entropy metric used in the proposed algorithm is defined by the ratio of assigned resources (r') per activity over the total resource units (rT) required for completing the project, while the overall project entropy value is related to the level of uniformity in the resource assignments: the higher the value of entropy is, the more ‘levelled’ the resource assignments are.

Resource levelling is a subset of resource-constrained scheduling problems (RCSP), stemming from the need to balance the use of resources over time and for resolving over-allocations or conflicts in their use. This need for balancing the workload of selected resources over the course of the project usually comes at the expense of time and/or cost. Among the most used heuristics for resource levelling is a method developed by Harris (1978) termed Minimum Moment Method, later included in the PACK method (Harris, 1990), computerized by Martinez and Ioannou (1993) and later extended by Hiyassat (2000, 2001) to account for multiple-resource levelling. The method recognizes that an optimally levelled project should have a resource-utilization histogram of rectangular shape so it attempts to transform a project’s original resource histogram into one of rectangular shape.

Entropy is a measure of how smooth the transformation is between different system states and is a metric of a system’s order and stability. Entropy (H_x) can mathematically be evaluated as the product of the probability mass function (p_x) of a variable x, times the logarithm of the inverse of the probability (eq. 1).

$$M_x = \sum_{i=1}^{n_t} \left[ (t_i r_t)(0.5r_t) \right]$$  (1)

The entropy-optimization approach has recently been applied to bid-unbalancing (Christodoulou, 2009a), disorder-based scheduling (Christodoulou et al., 2009b) and resource levelling (Christodoulou et al., 2009c). In all cases, the probability metric (Eq.1) equation was related to both the resource assignments and the resource requirements for each and all project activities, and then utilized in either minimizing or maximizing the resulting total entropy, depending on the problem to be solved.

In the case of resource levelling, the entropy-maximization method is used to examine “how many units of a required resource should be diverted to an activity in order to maximize its entropy, subject
to a limited overall resource availability within the examined time-period" (Christodoulou et al., 2009c) and takes the form of the following optimization equations:

$$\max (H_T) = \max \left\{ \sum_{j=1}^{n_r} \sum_{i=1}^{n_t} \left[ \frac{r_{i,j}}{r_{T,j}} \ln \left( \frac{r_{i,j}}{r_{T,j}} \right) \right] \right\}$$

(2)

subject to,

$$\sum_{i=1}^{n_t} \left( r_{i,j} \right) \leq \left( r_{T,j} \right) \quad \forall j$$

(3)

$$r_{ij} : \text{integer} ; \quad r_{ij} > 0 ; \quad i,j: \text{integer}$$

(4)

where $j$ is the resource-type index, $n_r$ is the number of different resource types used in the project, $r_{ij}$ is the number of units of resource type $j$ used on time unit $i$, $n_t$ is the number of total time-units in the project (i.e. the project duration) and $r_{T,j}$ is the total number of units of resource type $j$ used in the project.

The entropy-maximization, and associated entropy metric, are applied to a small example network and the results are compared to the ones obtained by the Minimum Moment Method. The entropy maximization method, further to computational improvements over the Minimum Moment Method, allows for activity stretching (or compressing) and provides resource-allocation solutions that show improvement over previous approaches (Christodoulou et al., 2009c). Future work entails multi-resource levelling and the development of heuristic algorithms that formalize the steps needed for implementing the method.

References


