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Free vibration of a magneto-electro-elastic toroidal shell

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In this study, earlier work on the vibration of a magneto-electro-elastic (MEE) cylinder (Buchanan 2003) and of a cylindrical shell (Annigeri et al. 2006) is extended to cover a thick-walled MEE toroidal shell. The approach taken makes use of the differential quadrature method (DQM), and is validated by comparing results with those from two previous studies given in the literature. New results are determined for thick-walled elastic toroidal shells, with and without magneto-electro effects. Conclusions are drawn about the significance of the interactive effect.

The toroidal shell is complete in the meridional and circumferential directions, θ and φ. By assuming the mode shape in the circumferential direction φ, the mathematical solution is reduced to the two dimensions of the cross-section, r and θ. Three different material idealizations are considered in this study, isotropic elastic, orthotropic elastic, and magneto-electro-elastic. For the various idealizations, a single layer of homogenized material is assumed for the entire shell.

The complete set of equations for the vibration of a MEE solid or shell has been presented in cylindrical coordinates by Buchanan (2003) and Annigeri et al. (2006). The equations are extended here to cover a thick-walled MEE toroidal shell, with the aid of the three-dimensional elasticity theory presented by Redekop (1992). The equations of motion have the form

$$\frac{\partial\sigma_{rr}}{\partial r}+\gamma_1\sigma_{rr}-\frac{1}{r}\sigma_{\theta\theta}-\gamma_2\sigma_{\varphi\varphi}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{1}{\gamma}\frac{\partial\sigma_{r\varphi}}{\partial\varphi}-\gamma_3\sigma_{r\theta}=\rho\frac{\partial^2 u}{\partial t^2};$$

$$\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}-\gamma_3\sigma_{\theta\theta}+\frac{\partial\sigma_{r\theta}}{\partial r}+\gamma_4\sigma_{r\theta}+\frac{1}{\gamma}\frac{\partial\sigma_{\theta\varphi}}{\partial\varphi}+\gamma_3\sigma_{\varphi\varphi}=\rho\frac{\partial^2 v}{\partial t^2};$$

$$\frac{1}{\gamma}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi}+\frac{1}{r}\frac{\partial\sigma_{\theta\varphi}}{\partial\theta}-2\gamma_3\sigma_{\theta\varphi}+\frac{\partial\sigma_{r\varphi}}{\partial r}+\gamma_5\sigma_{r\varphi}=\rho\frac{\partial^2 w}{\partial t^2};$$

$$\frac{\partial D_r}{\partial r}+\gamma_1 D_r+\frac{1}{r}\frac{\partial D_\theta}{\partial\theta}-\gamma_3 D_\theta+\frac{1}{\gamma}\frac{\partial D_\varphi}{\partial\varphi}=0;\frac{\partial B_r}{\partial r}+\gamma_1 B_r+\frac{1}{r}\frac{\partial B_\theta}{\partial\theta}-\gamma_3 B_\theta+\frac{1}{\gamma}\frac{\partial B_\varphi}{\partial\varphi}=0$$

Using constitutive and kinematic relations the equations are recast in terms of five basic unknowns; three elastic displacements, and magnetic and electric potentials. The solution for the numerical problem is found using the DQM. Enforcement of the governing equations and boundary conditions at sampling points leads to a set of simultaneous equations in terms of the values of the basic unknowns at the sampling points, and the unknown frequency ω. The results are given in terms of a non-dimensional frequency, $\Omega=\omega r_o\sqrt{\rho/C_{44}}$, where r_o is a geometric parameter, and ρ and C_{44} are material parameters.

For validation, comparisons are presented for two separate problems. For the first problem, results from a one-dimensional DQM solution for the natural frequencies of a MEE solid cylinder are compared in Table 1 with results given earlier by Buchanan (2003). There is rapid convergence in the DQM solution, and close agreement with the earlier results is obtained (maximum difference less than 3%). For a second validation problem, involving natural frequencies of vibration in an elastic toroidal shell, rapid convergence was also observed, and close agreement with the earlier results was again obtained (maximum difference less than 1%).

Table 1. Comparison of frequencies Ω for MEE* solid cylinder with results by Buchanan, 2003 (Table 5), 2nd circumferential harmonic, *k* refers to axial mode of vibration.

k	1.0			2.0			4.0		
	Buch.	DQM		Buch.	DQM		Buch.	DQM	
Mode		15	25		15	25		15	25
1	2.305	2.331	2.329	2.605	2.668	2.666	4.042	4.104	4.102
2	3.409	3.400	3.399	4.040	4.018	4.017	5.402	5.367	5.366
3	4.672	4.673	4.674	5.275	5.268	5.269	6.767	6.727	6.730
4	6.763	6.762	6.762	7.033	7.037	7.037	8.389	8.350	8.350

* C_{11}=4.36, C_{12}=2.4, $C_{13}=C_{12}$, C_{33}=4.3, C_{44}=1, C_{66}=0.98, e_{15}=0, e_{31}=-1/3; e_{33}=1, ε_{11}=0.35556, ε_{33}=5.15556, q_{15}=0.57971, q_{31}=0.76812, q_{33}=1, μ_{11}=-84.0159, μ_{33}=39.9076, m_{11}=0.000143, m_{33}=0.054493, ρ=1.

New results are presented for two problems. For the first problem, concerning the vibration of a thick-walled orthotropic elastic toroidal shell, the six lowest frequencies are presented for the 2nd harmonic for nine geometric cases, using elastic properties that correspond to the values of earlier work. In comparing the results for similar isotropic shells, it was seen that the changes in material properties led to only minor changes in the fundamental frequencies.

For the second problem concerning the vibration of a thick-walled MEE toroidal shell, the six lowest frequencies are presented for the same nine geometric cases considered in the first problem. The material properties considered now also include interactive effects. In comparing these results with the previous ones, it was seen that, for the range of geometric parameters considered, small changes in frequencies (<5%) are produced by the consideration of the interactive effects.

A semi-analytical solution has been presented to the vibration problem of a MEE thick-walled toroidal shell using the DQM. The current results show good agreement with earlier results. New results obtained for purely elastic shells and for MEE shells show minor differences in the computed natural frequencies in the range of the geometric parameters considered in the study.

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