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An entropy-based method for resource leveling

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Abstract

The paper revisits the Minimum Moment Method – a heuristic commonly used in resource levelling – and restates it as an Entropy Maximization problem. The proposed entropy-maximization method makes use of the general theory of entropy and two of its principal properties (subadditivity and maximality) to redefine resource levelling in terms of the entropy caused, and to develop an algorithmic framework for its solution. The entropy metric used in the proposed algorithm is defined by the ratio of assigned resources (r') per activity over the total resource units (r_T) required for completing the project, while the overall project entropy value is related to the level of uniformity in the resource assignments: the higher the value of entropy is, the more 'levelled' the resource assignments are. The entropy maximization method, further to computational improvements over the minimum moment method, allows for activity stretching (or compressing) and provides resource-allocation solutions that show improvement over previous approaches. A case study including finish-to-start, start-to-start and finish-to-finish relationships is also presented that validates the results.

Keywords: resource-levelling, entropy

1 Introduction

Resource-constrained scheduling problems (RCSP) are scheduling problems involving resources with limited capacity or of limited availability, constrained in their capacity to meet a constructor's predefined primary objectives for a timely and cost-efficient project completion. RCSP are NP-hard problems RCSPs are NP-hard problems, with analytical solutions increasingly more difficult to obtain as the size and complexity of the underlying schedule network increase. In such cases, solving the resource-constrained network can not be done analytically, but rather numerically or by use of heuristics which are utilized to obtain optimal or near-optimal solutions.

Common approaches to solving RCSP utilize implicit enumeration and backtracking (the methods used by most commercially available exact solvers) such as branch and bound methods, as proposed in Brucker and Knust (2003), Demeulemeester and Herroelen (1992, 1997), Mingozzi et al. (1998), Crawford (1996) and Garey et al. (1976). Other methods utilize intelligent branching and evaluation techniques such as mathematical programming (Winston and Venkataramanan 2002), dynamic programming (Carruthers and Battersby 1966), zero-one programming (Patterson 1984), or artificial agent techniques such as genetic algorithms (Hegazy 1999) and ant colony optimization (Christodoulou 2005, Christodoulou 2007). Also common are heuristic techniques, such as the ones proposed by Brucker et al. (1998) and Aslani (2007).

Resource levelling is a subset of RCSP, stemming from the need to balance the use of resources over time and for resolving over-allocations or conflicts in their use. This need for balancing the workload of selected resources over the course of the project usually comes at the expense of time and/or cost. Among the most used heuristics for resource levelling is a method developed by Harris (1978) termed Minimum Moment Method, later included in the PACK method (Harris, 1990), computerized by Martinez and Ioannou (1993) and later extended by Hiyassat (2000, 2001) to account for multiple-resource levelling. The method recognizes that an optimally levelled project should have a resource-utilization histogram of rectangular shape so it attempts to transform a project's original resource histogram into one of rectangular shape.

2 The Minimum Moment Method

In brief, as previously noted, the Minimum Moment Method assumes that the moment of the daily resource demands about the horizontal axis of a project's resource histogram is a good measure of the resource utilization and that the optimal resource allocation exists when the total moment is at a minimum, thus when the resource histogram is of rectangular shape. The method's objective is to reduce the daily fluctuations in resource demand by shifting activities in time and within each activity's free float so as to avoid impacting successor activities. This activity shifting is typically time-constrained (the overall project duration is expected to remain unchanged) but not necessarily resource-constrained. A detailed description of the underlying heuristic can be found in the original work by Harris (1978), as well as in the work by Martinez and Ioannou (1993), and Hegazy (2001). Extensions of the original method and further case studies can also be found in Harris (1990), Martinez and Ioannou (1992) and Hiyassat (2001). Mathematically, the total resource moment (M_x) can be obtained by summing up the individual resource moments about the time axis, and is given by

$$M_x = \sum_{i=1}^{n_t} \left[(t_i r_i)(0.5 r_i) \right] \tag{1}$$

where i is the time interval index, n_t is the number of time intervals comprising the resource histogram, and t_i, r_i are the time and resource values of the i^{th} histogram interval respectively.

The PACK method (Harris 1990) is based on the minimum moment method and attempts to 'pack' activities one by one so that their daily resource requirements fill the largest gaps in the daily resource histogram. Instead of shifting activities, the PACK method first builds a histogram considering only critical activities. The remaining activities are ordered in a processing queue based on daily resource requirement (in decreasing order), total float (in increasing order) and sequence step (in decreasing order). Activities are then hierarchically selected from the processing queue and positioned in time between the originally scheduled early start and late start time. The activity shift is chosen so that it minimizes the sum of the daily resource requirement and it considers the impact on the successor activities. In most cases, the result of the PACK method is the same as, or very close to, the minimum moment method (Martinez and Ioannou 1992).

2.1 Limitations of the Minimum Moment and PACK methods

One of the most notable limitations of the two methods is the assumption that the duration of each activity remains unchanged and that improvements in resource allocation can be achieved only through activity shifting and the utilization of each activity's free float. Furthermore, no consideration is given to possible daily resource constraints, necessitating the stretching (or compression) of affected activities over time. In the case of activity stretching, activities are allowed to 'stretch' in time while remaining continuous, while in the case of activity compression the resource assignments can be bigger than originally required thus shortening the corresponding activity durations.

3 Entropy maximization

Entropy is a measure of the unavailability of a system's energy to do work, and it is central to the second law of thermodynamics which deals with physical processes and the degree of spontaneity in their occurrence (spontaneous changes occur with an increase in entropy). Entropy is a measure of how smooth the transformation is between different system states and is a metric of a system's order and stability. Entropy (H_x) can mathematically be evaluated as the product of the probability mass function (p_x) of a variable x, times the logarithm of the inverse of the probability (eq. 2).

$$H_x = \sum_x p_x \ln\left(\frac{1}{p_x}\right) \tag{2}$$

Entropy's subadditivity and maximality properties are of particular interest. The former (subadditivity) states that the function's value for the sum of two elements is always less than or equal to the sum of the function's values for each element. The latter (maximality) states that the entropy function takes the greatest value when all admissible outcomes have equal probabilities. In other words, maximal uncertainty is reached for the equiprobability distribution of possible outcomes. As an illustration of these properties consider the case of an activity with duration of 3 days and requiring 6 resource units. Should we define the probability function used in the entropy equation as the ratio of allocated resources over the total resource requirements to complete the project, and assuming that a linear relationship between resource usage and resource productivity then, as Table 1 shows, the entropy is maximized when the resource moment is minimized.

Table 1. Resource-levelled example, with corresponding moment and entropy values.

Case	Daily Resource Usage						Resource	Activity	Resource	Entropy
	t=1	t=2	t=3	t=4	t=5	t=6	Constraint	Duration	Moment	
1	6	-	-	-	-	-	6	1	18.0	0.000
2	5	1	-	-	-	-	5	2	13.0	0.451
3	4	2	-	-	-	-	4	2	10.0	0.637
4	4	1	1	-	-	-	4	3	9.0	0.868
5	3	3	-	-	-	-	3	2	9.0	0.693
6	3	2	1	-	-	-	3	3	7.0	1.011
7	3	1	1	1	-	-	3	4	6.0	1.242
8	2	2	2	-	-	-	2	3	6.0	1.099
9	2	2	1	1	-	-	2	4	5.0	1.330
10	2	1	1	1	1	-	2	5	4.0	1.561
11	1	1	1	1	1	1	1	6	3.0	1.792

4 Resource-levelling through entropy-maximization

The entropy-optimization approach has recently been applied to bid-unbalancing (Christodoulou, 2009a), disorder-based scheduling (Christodoulou et al., 2009b) and resource levelling (Christodoulou et al., 2009c). In all cases, the probability metric (Eq.1) equation was related to both the resource assignments and the resource requirements for each and all project activities, and then utilized in either minimizing or maximizing the resulting total entropy, depending on the problem to be solved.

In the case of resource levelling, the entropy-maximization method is used to examine "*how many units of a required resource should be diverted to an activity in order to maximize its entropy, subject to a limited overall resource availability within the examined time-period*" (Christodoulou et al., 2009c) and takes the form of the following optimization equations:

$$\max\left(H_T\right) = \max\left\{\sum_{j=1}^{n_r}\sum_{i=1}^{n_t}\left[\frac{r_{i,j}}{r_{T,j}}\ln\left(\frac{r_{i,j}}{r_{T,j}}\right)\right]\right\} \quad (3)$$

subject to

$$\sum_{i=1}^{n_t}\left(r_{i,j}\right) \leq \left(r_{T,j}\right) \quad \forall j \quad (4)$$

$\boldsymbol{r_{i,j}}$ **: integer;** $\boldsymbol{r_{i,j} > 0}$**;** $\boldsymbol{i,j}$**: integer** (5)

where j is the resource-type index, n_r is the number of different resource types used in the project, $r_{i,j}$ is the number of units of resource type j used on time unit i, n_t is the number of total time-units in the project (i.e. the project duration) and $r_{T,j}$ is the total number of units of resource type j used in the project. The constraints in the above equations refer to the overall project resource-availability constraints (eq.4), the assumption that resource assignments are of integer value (fractional resource assignments are disallowed as non-physical assignments) (eq.5), and the exclusion of zero-value resource assignments for any of the project activities requiring resources to progress (eq.5).

5 Example

For demonstrating the method let us consider the project network shown in Fig. 1, utilizing not only finish-to-start relationships but also start-to-start and finish-to-finish relationships. The unconstrained network has a critical path consisting of activities B, F and K, for a total duration of 15 days. The corresponding unconstrained resource histogram shows resource demands as high as 17 units per day, for a total workload of 109 man-days. As per entropy's additivity and maximality properties, the entropy function takes the maximal value when all admissible outcomes have equal probabilities, resulting in an upper bound obtained by dividing the total man-days required to complete the project by the total project duration. For the network in study this value is at 109/15 = 7.27 units/day, for a total project entropy value of $H_T = 2.708$.

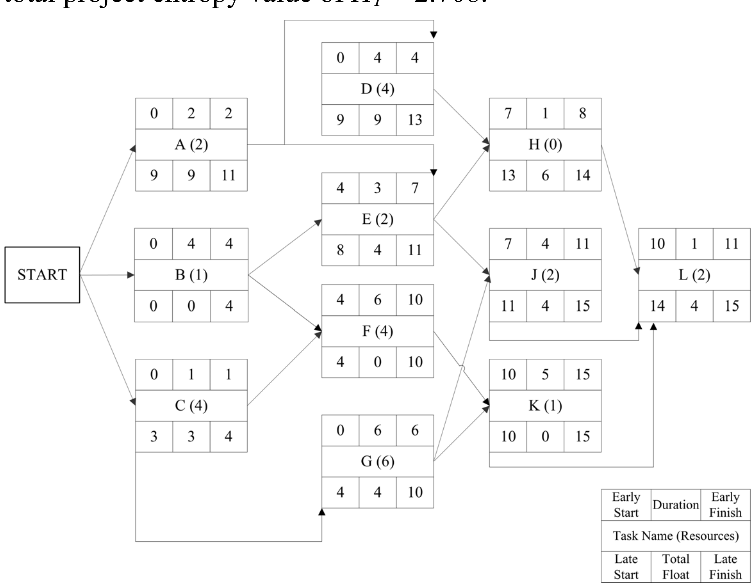

Figure 1. Example project network and resource requirements.

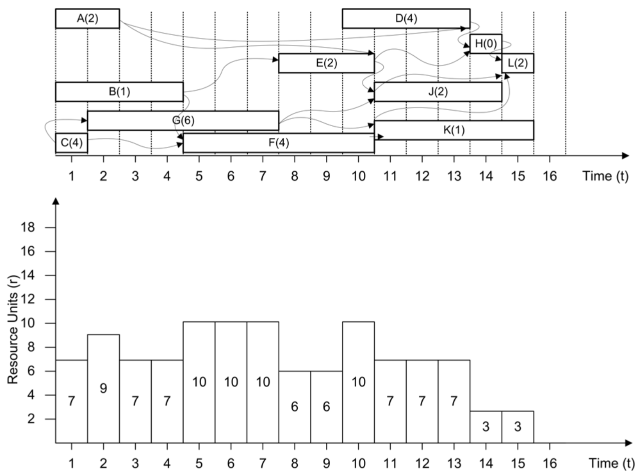

Figure 2. Resource levelling of the example network using the Minimum Moment Method.

If the Minimum Moment Method is used for resource-levelling the example network, the resource histogram shown in Fig. 2 can be obtained, for an entropy value of $H_T = 2.657$. This value is higher that the one for the unlevelled network ($H_T = 2.444$) indicating a higher optimization of resource usage.

Solution of the example network by use of the entropy method can be performed via simulation. The simulation, assuming that the project duration is constrained (as in the Minimum Moment Method) but that the individual activity durations are not, produces several feasible solutions with the best being for the following activity resource assignments: $\{r_A = 2, r_B = 2, r_C = 4, r_D = 2, r_E = 4, r_F = 5, r_G = 3, r_H = 0, r_J = 4, r_K = 2, r_L = 2\}$. The corresponding total project duration is 15 days, daily resource assignments of {9, 5, 8, 8, 8, 10, 9, 5, 5, 5, 5, 7, 10, 6, 7} (Fig. 3) and a corresponding entropy value of $H_T = 2.675$. In terms of the resource moment, the minimum moment method (Fig.2) produces a value of $M_x = 432.50$, while the entropy-maximization method (Fig. 3) produces a resource moment of $M_x = 406.50$. It can thus be seen that the entropy-maximization method (Fig. 3) results in a resource allocation that shows improvement over result obtained by the minimum moment method (Fig. 2).

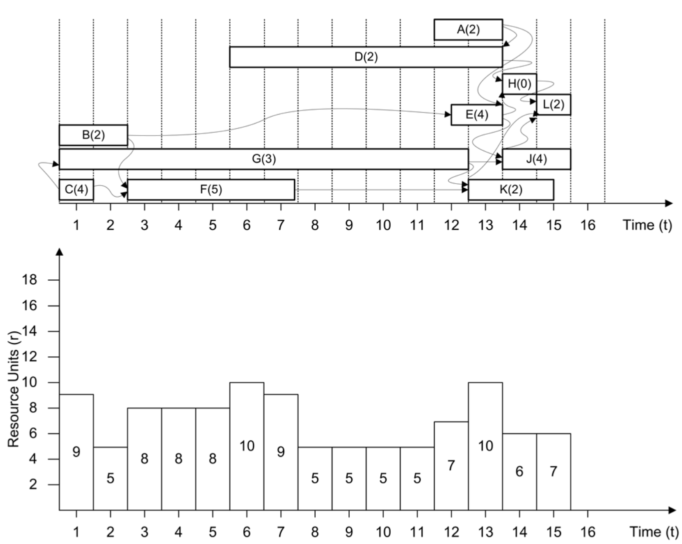

Figure 3. Resource levelling of the example network using the Entropy Maximization Method.

6 Conclusions

The entropy-maximization, and the entropy metric used, have shown to be a good alternative to the Minimum Moment Method for resource levelling. Unlike the minimum moment method, though, which does not allow for activity stretching (or compressing), the entropy maximization method accounts for such possibility and, if activity stretching is allowed, then the method provides resource allocation solutions that are even more optimized than the ones obtained from the minimum moment method (Christodoulou et al., 2009c). Future work entails multi-resource levelling and the development of heuristic algorithms that formalize the steps needed for implementing the method.

References

ASLANI, P., 2007. *Dynamic resource-constrained scheduling*. PhD thesis, Polytechnic University, Brooklyn, NY.

BRUCKER, P. and KNUST, S., 2003. Lower bounds for resource-constrained project scheduling problems. *European Journal of Operational Research*, 149, 302–313.

CARRUTHERS, J. A. and BATTERSBY, A., 1966. Advances in critical path methods. *Operations Research*, 17, 359–380.

CHRISTODOULOU, S., 2005. Ant colony optimization in construction scheduling. In: *ASCE International Conference on Computing in Civil Engineering, Cancun, Mexico*.

CHRISTODOULOU, S, 2007. Resource-constrained scheduling using ant colony optimization. In: *Ninth International Conference on the Application of Artificial Intelligence to Civil, Structural and Environmental Engineering (AICC07), St. Julians, Malta*.

CHRISTODOULOU, S., 2009a A bid-unbalancing method for lowering a contractor's financial risk. *Construction Management and Economics*, 26(12), 1291–1302.

CHRISTODOULOU, S., ELLINAS, G., and ASLANI, P., 2009b. Disorder considerations in resource-constrained scheduling. *Construction Management and Economics*, 27(3), 229–240.

CHRISTODOULOU, S., ELLINAS, G., and MICHAELIDOU-KAMENOU, A., 2009c. The Minimum Moment Method for Resource Leveling Using Entropy Maximization. *Construction Engineering and Management*, In Print (http://dx.doi.org/10.1061/(ASCE)CO.1943-7862.0000149).

CRAWFORD, J., 1996. An approach to resource constrained project scheduling. In: *Proc. 1996 AI and Manufacturing Research Planning Workshop*.

DEMEULEMEESTER, E. and HERROELEN, W., 1992. A branch-and-bound procedure for the multiple resource-constrained project scheduling problem. *Management Science*, 38, 1803–1818.

DEMEULEMEESTER, E. L. and HERROELEN, W. S., 1997. New benchmark results for the resource-constrained project scheduling problem. *Management Science*, 43, 1485–1492.

GAREY, M. R., GRAHAM, R. L., JOHNSON, D. S., and YAO, A. C. C., 1976. Resource constrained scheduling as generalized bin packing. *J. Comb. Th. Ser. A*, 21, 257–298.

HARRIS, R. B., 1978. *Precedence and arrow networking techniques for construction.* Wiley, New York.

HARRIS, R. B., 1990. Packing method for resource leveling (pack). *Journal of Construction Engineering and Management*, 116(2), 331–350.

HEGAZY, T., 1999. Optimization of resource allocation and leveling using genetic algorithms. *Journal of Construction Engineering and Management*, 125(3), 167–175.

HEGAZY, T. (2001). *Computer-Based Construction Project Management*. Prentice Hall.

HIYASSAT, M. A. S., 2000. Modification of minimum moment approach in resource levelling. *Journal of Construction Engineering and Management*, 126(4), 278–284.

HIYASSAT, M. A. S., 2001. Applying modified minimum moment method to multiple resource levelling. *Journal of Construction Engineering and Management*, 127(3), 192–198.

MARTINEZ, J. and IOANNOU, P., 1993. Resource leveling based on the modified minimum moment heuristic. *Computing in Civil and Building Engineering*. ASCE, 287–294.

MINGOZZI, A., MANIEZZO, V., RICCIARDELLI, S., and BIANCO, L., 1998. An exact algorithm for the resource-constrained project scheduling problem based on a new mathematical formulation. *Management Science*, 44, 714–729.

PATTERSON, J. H., 1984. A comparison of exact approaches for solving the multiple constrained resource, project scheduling problem. *Management Science*, 30, 854–867.

WINSTON, W. L. and VENKATARAMANAN, M., 2002. *Introduction to mathematical programming*. Pacific Grove, California: Thomson-Brooks/Cole, 4th edition.