An evolutionary numerical approach to modelling the mechanical properties of rubber concrete

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Abstract

Making use of the discarded rubber tire as the concrete aggregate may be considered as a solution to the environmental problems of disposal of this waste material. The addition of rubber aggregates in concrete decreases the mechanical properties of concrete depending mainly on the type and content of the rubber used. Also, new or modified models are required to describe the behaviour of the composite structure. In this research, a new data mining technique, named Evolutionary Polynomial Regression (EPR), is used to predict the mechanical behaviour of rubber concrete. Data used in training and testing of the EPR models were obtained from results of an experimental study involving a total of 70 rubber concrete samples. Three separate models were developed to predict the compressive strength, splitting tensile strength, and static elastic modulus of the concrete as a function of cement, silica fume, water, superplasticizer, coarse aggregate, fine aggregate, crumb rubber, and tire chips contents. It is found that the developed EPR models are able to predict various aspects of the behaviour of rubber concrete with high accuracy. The proposed models also showed perfect agreement with the experimental results, making the user able to estimate the mechanical properties of the rubber concrete.

Keywords: rubber concrete, mechanical behaviour, evolutionary modelling technique

1 Introduction

One of the most crucial environmental issues all around the world is the disposal of the waste materials. Accumulations of discarded waste tires have been a major concern because the waste rubber is not easily biodegradable even after a long period of landfill treatment. Innovative solutions to meet the challenge of tire disposal problem have long been under development. Producing new materials and energy using waste rubber are alternative ways to disposal of this type of waste (Atal et al. 1995). Because of high capital investment involved, using tires as a fuel for cement kiln is technically feasible but economically is not very attractive (Siddique et al. 2004). Thus, the conventional solution is to store them on empty land, which indirectly creates other problems because they become fire hazard or insect and animal habitation (Siddique et al. 2004, Sukontasukkul et al. 2006). Therefore, recycling of the waste tires is an attractive solution. There has been an interest in using the recycled waste tire products. In recent years, much research has been carried out in an attempt to reuse the abandoned tires by grinding them into small particles (crumb rubber or tire chips) and use in asphalts, sealants, and rubber sheets. Of particular interest has been the use of waste tires in Portland cement concrete (Sukontasukkul et al. 2006).
In recent years, considerable amount of research has focused on the use of tire rubber in the production of concrete. Eldin and Senouci (1993) investigated the strength and toughness properties of concrete containing two types of tire rubber. Khatip and Bayomy (1999) used recycled tire rubber as aggregate in the concrete mixtures with different rubber contents. The properties of the rubber concrete were also studied by Topcu (1995). Benazzouk et al. (2003) examined the physico-mechanical properties of cement–rubber composites by the use of two types of rubber aggregates, with the aim of developing a highly deformable material. All of the studies mentioned above have revealed that a general reduction in the basic engineering properties is observed when tire rubber is used in the concrete. Moreover, the reduction in the strength appears to be more remarkable with increasing the rubber content in the composite and the coarse grading of rubber aggregates lowers the strength more than the finer grading. Guneyisi et al. (2004) incorporated silica fume into the rubber concrete to diminish this strength loss that is resulted from the use of rubber aggregates. However, the silica fume had a positive effect on increasing the mechanical properties of the rubber concrete.

There is ample evidence that the strength values of the concretes decrease with the use of tire rubber in concrete. However, there exists no explicit formulation in literature to predict this strength loss. In recent years, soft computing techniques, especially artificial neural network (ANN) and genetic programming, have been applied to many engineering problems. ANN have been successfully used in concrete mix-design (Lee et al. 2003) estimating the shear capacity of deep beams (Sanad et al. 2001), predicting the tensile and shear capacity of anchors (Sakla et al. 2005, Alqedra et al. 2005, Gesoglu et al. 2007), etc. Similarly, genetic programming has been utilized in various engineering applications such as predicting the cement strength (Baykasoglu et al. 2004), optimum design of structures (Yang et al. 2002), and modeling of shear strength of RC deep beams (Ashour et al. 2003). Another research work has also used ANN and genetic programming to evaluate the strength loss of the concrete with tire rubber (Gesoglu et al. 2009).

The current paper introduces a new approach to predict the mechanical behaviour and strength loss of the concrete with tire rubber aggregates, using data from literature.

2 Evolutionary polynomial regression

Evolutionary polynomial regression (EPR) is a data-driven method based on evolutionary computing, aimed to search for polynomial structures representing a system. A general EPR expression can be presented as:

\[ y = \sum_{j=1}^{n} F(X, f(X), a_j) + a_0 \]  

where \( y \) is the estimated vector of output of the process; \( a_i \) is a constant; \( F \) is a function constructed by the process; \( X \) is the matrix of input variables; \( f \) is a function defined by the user; and \( n \) is the number of terms of the target expression. The general functional structure represented by \( F(X, f(x), a_j) \) is constructed from elementary functions by EPR using a Genetic Algorithm (GA) strategy. The GA is employed to select the useful input vectors from \( X \) to be combined. The building blocks (elements) of the structure of \( F \) are defined by the user based on understanding of the physical process. While the selection of feasible structures to be combined is done through an evolutionary process the parameters \( a_i \) are estimated by the least square method. Detailed description of the EPR method can be found in Rezania et al. (2008).
3 Database

In order to produce models to represent the mechanical behavior of rubber concrete, the data from an experimental study (Guneyisi et al. 2004) is used. In that study, a test program was carried out to provide information about the compressive strength, splitting tensile strength and static elastic modulus of the rubber concrete with and without silica fume. Two types of tire rubber (crumb rubber and tire chips) were used as fine and coarse aggregates in the production of rubber concrete mixtures. Six different rubber contents varying from 2.5% to 50% by total volume of aggregate were used. The concrete samples with silica fume were produced by partial replacement of cement with silica fume at varying amounts of 5–20%. In total, 70 concrete mixtures were tested out of which, a set of data from 56 mixtures was used to train the models while the remaining data were used for testing and assessing the generalisation capabilities of the models. The data were randomly selected to generate both the training and testing sets.

4 EPR Procedure

In the evolutionary process of building EPR models, a number of constraints can be implemented to control the output models in terms of the type of functions used, number of terms, range of exponents, number of generations etc. In this process there is a potential to achieve different models for a particular problem which enables the user to gain additional information for different scenarios (Rezania et al., 2008). Applying the EPR procedure, the evolutionary process starts from a constant mean of output values. By increasing the number of evolutions it gradually picks up the different participating parameters in order to form equations describing the relationship between the parameters of the system. Each proposed model is trained using the training data and tested using the testing data provided. The level of accuracy at each stage is evaluated based on the coefficient of determination (COD) i.e., the fitness function as:

\[
COD = 1 - \frac{\sum_{i=1}^{N} (Y_a - Y_p)^2}{\sum_{i=1}^{N} (Y_a - \frac{1}{N} \sum_{i=1}^{N} Y_a)^2}
\]  

(2)

where \(Y_a\) is the actual input value; \(Y_p\) is the EPR predicted value and \(N\) is the number of data on which the COD is computed. If the model fitness is not acceptable or the other termination criteria (in terms of maximum number of generations and maximum number of terms) are not satisfied, the current model should go through another evolution in order to obtain a new model.

5 EPR models

From the total of 70 cases in the database, 56 cases are used to develop the EPR models and the remaining cases are used as unseen cases to validate the developed models. Among the resultant equations developed using EPR the ones with the highest value of coefficient of determination (COD) are selected for compressive strength (\(f'_c\)), splitting tensile strength (\(f_{sp}\)), and elastic modulus of rubber concrete (\(E_c\)) as:

\[
f'_c = \frac{3.7 \times 10^{-9} \cdot SP^3 \cdot CA^1}{C} - 1.18 \times 10^{-10} \cdot FA^1 \cdot CR^2 + \frac{0.014SF^2 \cdot CA^3}{C^3 \cdot SP^3 \cdot FA} + 7.67 \times 10^{-11} \cdot SF \cdot W \cdot SP \cdot FA^2 - 0.00036SF^2 + 3.23 \times 10^{-6} \cdot CA^2 \cdot CR^2 - 0.487C + 52.3
\]

(3)
\[ f_c = \frac{2.21 \times 10^{-15} CR^3 \cdot TC^3 + 0.02CA^2 \cdot FA}{W^3} + \frac{2.72 \times 10^{-8} W^2 \cdot SP \cdot FA^3}{C^2 \cdot CA} + 0.097 \]  

(4)

\[ E_c = -0.3C + \frac{635.82 SP}{FA} + \frac{4.74 \times 10^{-6} C^3 \cdot SF^2 \cdot SP \cdot CR^3}{FA^2 \cdot TC^2} + \frac{4.961CA \cdot CR^2}{W^3} - \frac{42234.25TC}{W^2} + 353.223 \]  

(5)

where C, SF, W, SP, CA, FA, CR, and TC are cement, silica fume, water, superplasticizer, coarse aggregate, fine aggregate, crumb rubber, and tire chips contents respectively.

Figure 1 shows the comparison of the results of the EPR models with the experimental for the training and testing data sets.

A very close agreement can be seen between the model predictions and the experimental data. Coefficient of determination is used to compare the results of the EPR models with those obtained from linear regression, artificial neural network and genetic programming techniques (after Gesoglu et. al 2009). Comparison of the results shows that the EPR models are able to predict the behavior of rubber concrete with a high accuracy.

Table 1. COD values(%) for LR, ANN, GEP and EPR models based on testing data

<table>
<thead>
<tr>
<th>Model</th>
<th>COD values ((f_c))</th>
<th>COD values ((f_{sp}))</th>
<th>COD values ((E_c))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Regression</td>
<td>86.89</td>
<td>79.57</td>
<td>84.75</td>
</tr>
<tr>
<td>Genetic Programming</td>
<td>98.18</td>
<td>98.22</td>
<td>98.87</td>
</tr>
<tr>
<td>Artificial Neural Network</td>
<td>99.94</td>
<td>98.69</td>
<td>98.58</td>
</tr>
<tr>
<td>(ANN)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Evolutionary Polynomial</td>
<td>98.36</td>
<td>97.49</td>
<td>98.68</td>
</tr>
<tr>
<td>Regression (EPR)</td>
<td></td>
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</tbody>
</table>
One of the main advantages of EPR is that it provides the relationship between the parameters in the form of a polynomial expression, so parametric studies can be conducted to investigate the contribution of different parameters involved. This advantage gives the opportunity to the user to gain an insight into the influence that different additives can have on the resultant concrete mixture. To do this, a sensitivity analysis has been carried out to investigate the effect of different contributing parameters on the behavior of rubber concrete. The usual approach to sensitivity analysis is adopted in which all parameters are set to their mean values except the one being changed in the range of its minimum and maximum values in the dataset. Results of the parametric study on the compressive strength of the rubber concrete are shown in Figure 2. It is shown that increasing the amount of fine grained aggregates and tire chips (Figure 2a and b) decreases the compressive strength of the mixture, but any increase in the coarse grained aggregate content (Figure 2c) improves the compressive strength of the rubber concrete.

![Figure 2](a)  
(a)  

![Figure 2](b)  
(b)  

![Figure 2](c)  
(c)  

Figure 2. Results of the parametric study conducted on the EPR compressive strength model for rubber concrete

### 6 Discussion and conclusion

In recent years the use of pattern recognition methods such as ANN has been introduced as method for analysis of behaviour of rubber concrete based on experimental data. These methods have the advantages that they do not require any simplifying assumptions in developing the model. However, the neural network based models also suffer from a number of shortcomings including (i) the black box nature of the models and their inability to present an explicit relationship between the input and output parameters; (ii) the fact that they require the structure of the network (e.g., number of hidden layers, number of neurons in hidden layers, transfer functions, etc) to be identified a priori; and (iii) that the optimum structure and parameters of the network are obtained by a trial and error.

In this paper, a new approach was presented for modelling and analysis of the compressive strength, tensile splitting strength, and elastic modulus of rubber concrete using EPR. Three separate EPR models were developed and validated using a database of test data on characteristics of rubber concrete. The results of model predictions were compared with test data as well as results from linear regression, genetic programming, and neural network models. A parametric study was also conducted to evaluate the effects of different parameters on compressive strength of rubber concrete, and the
extent to which the developed models can represent the physical relationships between contributing parameters. The results show that the developed EPR models provide very accurate predictions for strength parameters of rubber concrete. The developed models present a structured and transparent representation of the system, allowing a physical interpretation of the problem that gives the user an insight into the relationship between the strength parameters and various contributing parameters. From practical point of view, the EPR models presented in this paper are easy to use and provide results that are as accurate and in some cases more accurate than the existing models.

An interesting feature of EPR is in the possibility of getting more than one model for a complex phenomenon. A further feature of EPR is the high level of interactivity between the user and the methodology. The user physical insight can be used to make hypotheses on the elements in the function $F$ and on its structure (Eq. 1). Another major advantage of the EPR approach is that as more data becomes available, the quality of the prediction can be easily improved by retraining the EPR model using the new data. However, it should be noted that the EPR models should not be used for extrapolation i.e. for new cases where one or more parameters fall outside the range of the parameters used in training, the predicted results should be taken with caution and allowance should be made for the uncertainty.

References


