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Finite element analysis of three dimensional shallow foundation using artificial intelligence based constitutive model

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Abstract

In this paper, a new approach is presented for constitutive modelling of materials in finite element analysis. The proposed approach provides a unified framework for modelling of complex materials using evolutionary polynomial regression (EPR). A procedure is presented for construction of EPR-based constitutive model (EPRCM) and its integration in finite element procedure. The main advantage of EPRCM over conventional and neural network-based constitutive models is that it provides the optimum structure for the material constitutive model representation as well as its parameters, directly from raw experimental (or field) data. It can learn nonlinear and complex material behaviour without any prior assumption on the constitutive relationships. The proposed algorithm provides a transparent relationship for the constitutive material model that can readily be incorporated in a finite element model. The developed EPRCM-based finite element model is used to analyse a 3D shallow foundation and the results are compared with conventional methods. It is shown that the proposed approach provides an efficient alternative to conventional constitutive modelling in finite element analysis.

Keywords: constitutive modelling, evolutionary computation, data mining, finite element.

1 Introduction

Finite element method has, in recent years, been widely used as a powerful tool in the analysis of engineering problems. In this numerical analysis, the behaviour of the actual material is approximated with that of an idealized material that deforms in accordance with some constitutive relationships. Therefore, the choice of an appropriate constitutive model that adequately describes the behaviour of the material plays an important role in the accuracy and reliability of the numerical predictions. During the past few decades several constitutive models have been developed for various materials. Most of these models involve determination of material parameters, many of which have little or no physical meaning (Shin and Pande, 2000). Despite considerable complexities of constitutive theories, due to the erratic and complex nature of some materials such as soils, rocks, composites, etc., none of the existing constitutive models can completely describe the real behaviour of these materials under various stress paths and loading conditions.

In the past few decades, the use of artificial neural networks (ANN) has been introduced as an alternative approach to constitutive modelling of materials. The application of ANN for modelling the behaviour of concrete was first proposed by Ghaboussi et al. (1991). Ghaboussi & Sidarta (1998) presented an improved technique of ANN approximation for learning the mechanical behaviour of drained and undrained sand. The role of ANN in constitutive modelling was also studied by a number of other researchers (Ellis et al., 1995; Zhu et al., 1998; Javadi et al., 2003; 2004; 2009). These works indicated that neural net-work based constitutive models can capture nonlinear material behaviour with a high accuracy. It has been shown that the neural network-based constitutive models can be incorporated in finite element (or finite difference) codes (eg, Shin and Pande, 2000, Hashash et al. 2004, Javadi et al., 2003, 2009).

Although it has been shown by various researchers (including the authors) that the ANNs have a great potential in modelling of material behaviour, it is generally accepted NNCMs also have a number of drawbacks (Javadi & Rezania 2009). One of the main disadvantages of the NNCM is that the optimum structure of the ANN (such as number of inputs, number of hidden layers, transfer functions, etc.) must be identified a priori which is usually done through a time consuming trial and error procedure. Another major drawback of the NNCM approach is the large complexity of the network structure as it represents the knowledge in terms of a weight matrix together with biases that are not accessible to user. The lack of interpretability of ANN models has inhibited them from achieving their full potential in real world problems (Javadi & Rezania 2009).

This paper presents a new approach for constitutive modelling using EPR that overcomes the shortcomings of the ANN-based approach. In the proposed approach the optimum structure for the material constitutive model representation and its parameters are determined directly from raw data. Furthermore, it provides a transparent and structured representation of the constitutive relationships that can be readily incorporated in a finite element code. Although some work has been done in literature in the field of application of intelligent finite element method in engineering problems, however almost all of them were limited to 2D planar problems. In this paper the application of intelligent FEM in analysis of a 3D problem (a 3D shallow foundation) has been presented and the results have been compared to the other conventional material models. Moreover in this paper the implementation of the presented methodology in a well known general-purpose finite element code (ABAQUS) has been introduced. In what follows, the main principles of EPR will be outlined. The application of EPR in modelling of nonlinear constitutive relationships and the implementation of the obtained models in FE analysis will be illustrated with an example.

2 Evolutionary polynomial regression

Evolutionary polynomial regression (EPR) is a data-driven method based on evolutionary computing, aimed to search for polynomial structures representing a system. A general EPR expression can be presented as (Giustolisi & Savic 2006)

$$y = \sum_{j=1}^{n} F(X, f(X), a_j) + a_0 \quad \textbf{(1)}$$

where y is the estimated vector of output of the process; is a constant; *F* is a function constructed by the process; *X* is the matrix of input variables; *f* is a function defined by the user; n is the number of terms of the target expression. The general functional structure represented by *F* is constructed from elementary functions by EPR using a GA strategy. The GA is employed to select the useful input vectors from *X* to be combined. While the selection of feasible structures to be combined is done through an evolutionary process the parameters are estimated by the least square method.

3 EPR for constitutive modelling

In constitutive modelling using EPR, the raw experimental or in-situ test data are directly used for training the EPR model. In this approach, there are no mathematical models to select and as the EPR learns the constitutive relationships directly from the raw data it is the shortest route from experimental research to numerical modelling. There are no material parameters to be identified and as more data become available, the material model can be improved by re-training of the EPR using the additional data. Furthermore, the incorporation of an EPR in a finite element procedure avoids the need for complex yield/failure functions, flow rules, etc. An EPR equation can be incorporated in a finite element code/procedure in the same way as a conventional constitutive model. It can be incorporated either as incremental or total stress-strain strategies.

3.1 Input and output parameters

The choice of input and output quantities is determined by both the source of the data and the way the trained EPR model is to be used. A typical scheme to train most of the neural network based material models involves an input set providing the network with the information relating to the current state units (e.g., current stresses and current strains) and then a forward pass through the neural network yields the prediction of the next expected state of stress and/or strain relevant to an input strain or stress increment (Ghaboussi et al 1998). The same idea has been utilized in this work. Thus the mean stress p', deviatoric stress q, volumetric strain ε_v and distortional strain ε_q are used as the input parameters representing the current state of stress and strain in a load increment *i*, and the devatoric stress corresponding to the input incremental deviatoric strain $\Delta\varepsilon_q$ is used as the output parameter. The database is divided into two separate sets. One set is used for training to obtain the EPR model and the other one is used for validation to appraise the applicability of the trained model.

4 Intelligent finite element method

The developed EPRCMs are implemented in the widely used general-purpose finite element code ABAQUS through its user defined material module (UMAT). UMAT updates the stresses and provide the material Jacobian matrix for every increment in every integration point. In the developed methodology, the EPRCM replaces the role of a conventional constitutive model. The source of knowledge for EPR is a set of raw experimental (or in situ) data representing the mechanical response of the material to applied load. When EPR is used for constitutive description, the physical nature of the input–output data for the EPR is determined by the measured quantities, e.g., stresses, strains, etc. The manner in which EPRCM is incorporated in ABAQUS is depicted in Figure 1.

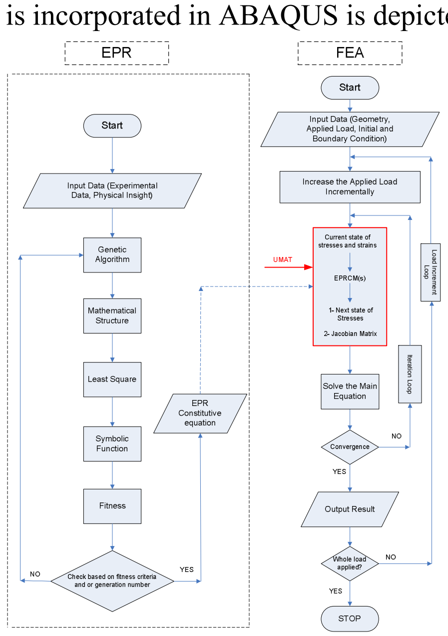

Figure 1. The incorporation of EPR-based material model in ABAQUS finite element software

In the EPR-based finite element procedure, during each load increment the Young's modulus, E_{EPR} is calculated from the EPR relationship between the increments of relevant stress and strain and used to form the element stiffness matrix (**D**). The stiffness matrix is then updated for every single element. Consequently the global stiffness matrix for a particular problem is assembled. The whole procedure ensures that the constitutive model follows the actual behaviour of the material, both at the element level and the global level.

5 Numerical example

To illustrate the developed computational methodology and its ability to analyze intricate and realistic boundary value problems, a numerical example of application of the developed intelligent finite element method to a geotechnical problem is presented. In this example, the application of the methodology for settlement of a three dimensional shallow foundation is examined.

The example involves analysis of a square shallow foundation subjected to applied displacement. Due to symmetry only a quarter of the domain is analyzed. The geometry of the foundation and the finite element mesh are shown in figure 2 (Helwany, 2007).

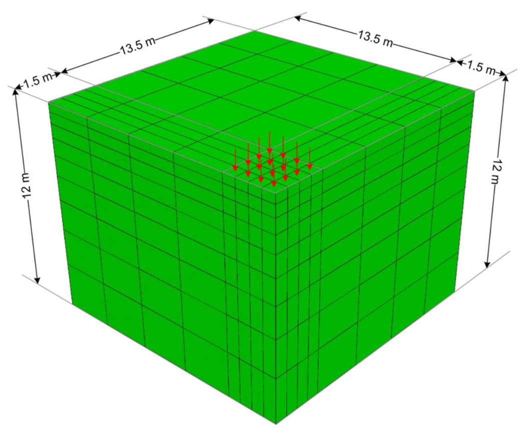

Figure 2. Dimension and finite element mesh of a quarter of the square shallow foundation

The finite element mesh includes 576 eight-node elements and 810 nodes. The aim of the analysis is to calculate bearing capacity of 3 m×3 m foundation on a 12 m thick homogeneous layer of a soil, using Mohr-Coulomb and EPRCM based finite element method. The foundation is situated at a depth of 0.38 m from ground level. A displacement of 50cm is imposed on footing as shown in figure 2.
The results from a series of triaxial tests were used in this example for the training of the EPR based constitutive model with an incremental stress-strain (tangential stiffness) strategy. It was assumed that the soil tested was representative of the material of the soil around the foundation. The test data were arranged as shown in Table 1 and used to train an EPRCM to model the stress-strain relationship for the soil. The results from 3 tests conducted at confining pressures of 100, 150 and 350 kPa were used for training of the EPR model while those for the fourth test at the confining pressure of 250 kPa were used for validation of the trained EPR model. At the end of the training and testing procedure, the selected best EPR model representing the behaviour of the soil is:

$$
\begin{aligned}
q^{i+1} = & -\frac{4867.5q^2}{p^3} + 1.22q + 4.42\times10^{-7}q^3 - \frac{7537.74\varepsilon_v^2}{\varepsilon_q^2 p^2} - 0.000662q^2 \\
& + \frac{0.17259\,\varepsilon_q^3\,\Delta\varepsilon_q}{\varepsilon_v^3 q^2} - \frac{0.063\varepsilon_q^2}{\varepsilon_v^2 q} + \frac{0.0021\varepsilon_q\, p^2}{q} + \frac{3042.6}{p} - 7.9324
\end{aligned}
\qquad \textbf{(2)}
$$

Table 1. Input and output parameters used for training the EPR constitutive model.

Input parameters					Output parameters
p'^{i}	q^{i}	ε_v^{i}	ε_q^{i}	$\Delta\varepsilon_q$	q^{i+1}

Figure 3a shows the results of the training of EPR model. It is clearly seen that, the EPR was able to capture the constitutive (nonlinear) stress-strain relationship for the soil with very good accuracy. The generalization capability of the EPRCM is shown in Figure 3b. The data from the test conducted at the confining pressure of 250kPa (which did not form a part of the training data) were used to test the trained EPRCM. The predicted output values of the EPR model are compared with the experimentally measured values in Figure 3b. It is seen that the generalization capability of the trained EPRCM is excellent. This shows that the EPR model was trained sufficiently to adequately model the stress-strain behaviour of the soil. The trained EPRCM was incorporated in the intelligent finite element (EPR-FEM) using UMAT in ABAQUS. The intelligent FE incorporating the EPR model was then used to simulate the behaviour of the foundation under applied displacement. For the conventional finite element analyses, the results of the triaxial tests were used to derive the material parameters for the Mohr-Coulomb model for the soil (see Table 2).

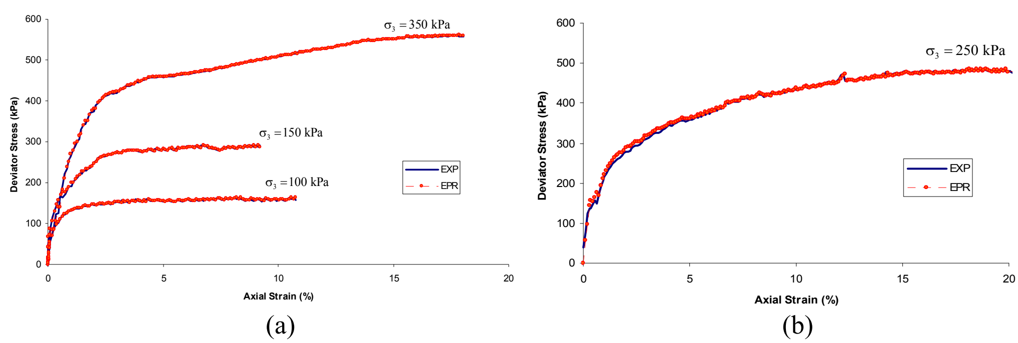

Figure 3. (a) Results of training of the ANN, (b) stress-strain relationship predicted by the trained EPR

Table 2. Material parameters for Mohr-Coulomb model

c' (kPa)	ϕ'(°)	ν	γ (kN/m^3)
8	30	0.33	18.14

Figure 4 shows the pressure-settlement curves for the centre of the foundation predicted by standard finite element analysis using the Mohr-Coulomb elasto-plastic model as well as the intelligent finite element method where the raw data from the triaxial tests were directly used in deriving the EPR-based constitutive model. In the initial elastic zone, the predictions of the two models are almost the same. As loading progresses, inelastic deformations start and differences appear in the predicted modes of pressure-settlement behaviour of the foundation. This could be due to the idealisations adopted in the conventional Mohr Coulomb model. It can be argued that the intelligent FE results are more reliable, as this method used the original raw experimental data (representing the stress-strain behaviour of the soil) to learn the constitutive relationships for the material and it did not assume a priori any particular constitutive relationships, yield conditions, etc.

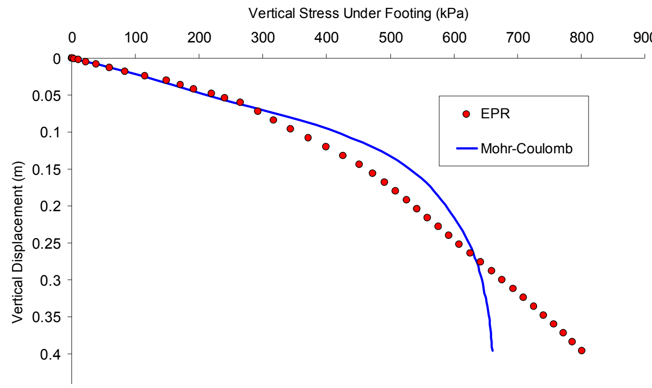

Figure 4. Comparison of results for foundation settlement, obtained from conventional FEA and intelligent FEA

6 Conclusion

An intelligent finite element method (EPR-FEM) has been developed based on the integration of an EPRCM in a finite element framework. In the developed methodology, the EPRCM is used as an alternative to the conventional constitutive models for the material. A procedure is presented for computing the stiffness matrix using the trained EPR model and incorporation of the EPRCM in a commercial finite element code ABAQUS. The efficiency of the proposed method has been demonstrated by successful application to a boundary value problem. The results of the analysis have been compared to those obtained from conventional FE analyses using the elastic-plastic models. The result shows that EPRCM can be successfully implemented in a finite element model as an effective alternative to conventional material models.

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