Nonlinear element model and damage estimating model of RC structures with arbitrary cross-section columns

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Abstract

The mechanics performance of RC structures will be weakened due to damage. A nonlinear element model and damage estimating model of RC frames are presented. The element model takes into account both material nonlinearity and geometric nonlinearity. The derived element stiffness matrices are suitable for RC structures with arbitrary cross-section members. The damage estimating model based on thermodynamic theory can ascertain structural damage assessment and provide safety evaluation. With the derived element model and damage estimate model, the nonlinear performance and cumulated damages are analyzed for RC L shaped columns structures. The analytical and damage estimate results corresponds well with the cumulated damage state of special columns structures under cycle loading.

Keywords: RC structures with special columns, nonlinear element model, damage estimating model

1 Introduction

The nonlinear analysis of reinforced concrete structures can reflect the structural stress state; it is the inevitable trend for engineering structure analysis. Advantages of the analysis method include that can reflect the structural stiffness degradation due to plastic internal force redistribution, the weak storey position, optimization structural design, and providing theoretical basis for practical engineering. Nonlinear finite element models for truss structure are divided into micro- and macro-model, and separate element is a kind of micro-element model (Jumin et. al., 1993). Due to considering the bond-slip between reinforcement and concrete, this kind of element is widely applied in structural analysis, but unfit for nonlinear analysis of engineering structures, because of discrete structural size must be small enough, which cause very large number of elements, large amount of calculation (Hongbiao and Jumin, 1990). Macro element model mainly include combined-element and integral-element, which are applied to analyze beam, column, wall of structure members. Advantages of macro-element model are simple calculation and fast, and it is applied to macro mechanical properties of structure.

Under strong earthquake, concrete structure may lead to large plastic deformation, serious damage and even collapse. Therefore, according to the structural future seismic damage prediction values, safety and reasonable structure seismic design should be used in engineering structures. Due to the complexity of earthquake ground motion and influence factors of diversity for structure damage, satisfied damage evaluation model is still not proposed. Code for seismic design of buildings is still the strength design and deformation checking method to realize the 3-level design principles, namely small earthquake not bad, middle earthquake repairable, and strong earthquake not collapse.
In order to realize the seismic design method based on damage, one of key question is to build damage estimating model applied for RC structure which is convenient for engineering application and can truly reflect damage level. Many scholars in countries had done many relevant research works in these aspect, their research results mainly included cumulative damage model for members and structures of reinforced concrete, the former researched local cumulative damage of structure (Banon et. al., 1981; Stephal and Yao, 1987; Wang, and Shan, 1987; Chung et. al., 1987; Gosain et. al., 1977), and the latter studied integral cumulative damage (Kratzig and Meskuris, 1987; Park and Ang, 1985; Bo and Jinping, 1993; Roufaiel and Meyer, 1987; Chung et. al., 1989; Jinping et. al., 1999; Yueqian et. al., 2006). However, local damage of structure under strong earthquake doesn't mean collapse of integral structure; further analyzed structural integral cumulative damage will have the reference value for earthquake damage prediction, structural reinforcement and repair. Due to the concrete structure with strong nonlinear characteristics and cumulative damage under earthquake, but research of the structural integral damage estimating model is still in the initial stage. Roufaiel (Roufaiel and Meyer, 1987) employed the relative displacement of the top floor to build structural cumulative damage model, Chung (Chung et. al., 1989), based on Park model, frame structure damage model with weighting coefficients method is suggested by Ou Jinping (Jinping et. al., 1999; Bo and Jinping, 1993), Liang Wenxing (Yueqian et. al., 2006).

In this paper, the RC beam-column element model is derived based on principle of virtual work and material constitutive laws, which consider material nonlinearity and geometry nonlinearity. The element is integrated around the reinforced concrete cross-section and along the beam-column elemental axis with double Gauss integration method, which makes the RC nonlinear beam-column element integration simple and accurate. The nonlinear analysis program of RC structures with arbitrary cross-sections is developed, based on the derived elemental model and integrated methods. A damage estimate model of RC structure based on thermodynamic theory is derived, and it is assumed that the initial scalar is the work done under ideal non-damage state. With the derived damage estimate model, the cumulated damages are analyzed for RC special columns frame structure according to experimental data.

2 Nonlinear beam-column element model with arbitrary cross-section

2.1 Analysis of RC cross-section

2.1.1 Strain plane of cross-section

In reinforced concrete beams, it is assumed: (1) plane section remains plane, (2) no bond slip between reinforcement and concrete; (3) cross-section deformed always keeps vertical with beam axis. According to the plane section assumption, in Cartesian coordinate system, the strain at any point \( A(y, z) \) of the cross-section (Bo et. al., 2000) is,

\[
\epsilon = \epsilon_0 + z\phi_y - y\phi_z = \hat{y}^T \hat{\epsilon}
\]

(1a)

And its increment form, \( d\epsilon = \hat{y}^T d\hat{\epsilon} \)

(1b)

where \( \hat{y}^T = [1 \ z \ -y] \) is the coordinates; \( \hat{\epsilon} = [\epsilon_0 \ \phi_y \ \phi_z]^T \) is the strain plain of the cross-section, \( \epsilon_0 \) is the strain at coordinate original point, \( \phi_y \) and \( \phi_z \) means curvatures of the cross-section around \( y \) and \( z \) axis respectively.

2.1.2 Stiffness matrix of cross-section

According to material strength theory, equilibrium equation of reinforced concrete arbitrary section under bi-eccentric compressive loading is as following,
\[
\begin{bmatrix}
N \\
M_y \\
M_z
\end{bmatrix} = \begin{bmatrix}
\int_{A_c} \sigma_c(\varepsilon) \, dA + \sum_{i=1}^{n} \sigma_{u,i} A_i \\
\int_{A_c} \sigma_c(\varepsilon) z \, dA + \sum_{i=1}^{n} \sigma_{u,i} z_i A_i \\
-\int_{A_c} \sigma_c(\varepsilon) y \, dA - \sum_{i=1}^{n} \sigma_{u,i} y_i A_i
\end{bmatrix}
\]

(2)

Where, \(N\) is normal force, \(M_y\) and \(M_z\) is bending moments around \(y\) and \(z\) axis, \(\sigma_c(\varepsilon)\) is concrete stress and \(A\) is concrete compressive and uncracking tensile area, \(\sigma_{u,i}, z_i, y_i\) are stress, area and coordinates of \(i\)'th reinforcement bars respectively.

The incremental form of the Eq. (2) is expressed in matrix,

\[
d\ddot{\sigma} = \begin{bmatrix} D_{l(c)} + D_{l(s)} \end{bmatrix} d\ddot{e} = D_0 d\ddot{e}
\]

(3)

Where, \(\{d\ddot{\sigma}\} = [dN \ dM_y \ dM_z]^T\), \(d\ddot{e} = [d\varepsilon_0 \ d\varphi_y \ d\varphi_z]^T\), \(D_{l(c)} + D_{l(s)}\) is the tangent stiffness of cross-section, \(D_{l(c)}\) is the contribution of concrete to tangent stiffness (or concrete stiffness in short), \(D_{l(s)}\) is the contribution of reinforcement bars. \(D_{l(s)}\) may be easy calculated in Cartesian coordinate system. \(D_{l(c)}\) related to the section shape, loading case and concrete constitutive laws, holds the main calculating work in section nonlinear analysis. The concrete stiffness is integrated by Gauss Integration (Bo et. al., 2000). The concrete matrix \(D_{l(c)}\) may be expressed as,

\[
D_{l(c)} = \begin{bmatrix}
\int_{A_c} f'(\varepsilon) dA_c & \int_{A_c} f'(\varepsilon) z dA_c & -\int_{A_c} f'(\varepsilon) y dA_c \\
\int_{A_c} f'(\varepsilon) z dA_c & \int_{A_c} f'(\varepsilon) z^2 dA_c & -\int_{A_c} f'(\varepsilon) z y dA_c \\
-\int_{A_c} f'(\varepsilon) y dA_c & -\int_{A_c} f'(\varepsilon) z y dA_c & \int_{A_c} f'(\varepsilon) y^2 dA_c
\end{bmatrix}
\]

(4)

2.2 RC beam-column element model

Consider a three-dimensional nonlinear system subjected to a concentrated force \(P\). The system is initially in a static equilibrium and then it subjects to a small increment of deformation. According to the principle of virtual displacement, the increments of the internal potential energy are equal to the external work done, i.e.,

\[
\delta w^T P = \iint \delta \varepsilon^T \sigma \, dv.
\]

(5)

Substituting strain-displacement relation and constitutive relation to the right side of Eq. (5), the equilibrium equation for the beam-column element is obtained as,

\[
P = \int_{L} B^T \ddot{\sigma} \, dx
\]

(6)

Where \(B\) is the geometric nonlinearity matrix of beam-column element when the higher order effects is involved, and \(\ddot{\sigma}\) is the cross-section forces as Eq.(3). In order to find the tangent stiffness matrix, the incremental form of the above equation with respect to the nodal displacement increment.

\[
dP = \int_{L} B^T \ddot{\sigma} \, dx + \int_{L} dB^T \ddot{\sigma} \, dx
\]

(7a)

When geometric matrix is nonlinear \(B(w)\) and constitutive law is nonlinear, Eq.(7a) becomes,

\[
dP = K_i \delta w
\]

(7b)
Where \( w \) is displacement vector of elemental nodes, \( K_t = \int_1 B_i^T D B_i \, dx \) is the tangent stiffness matrix, \( B_i(w) \) is the geometric nonlinear tangent matrix, \( D \) is the tangent stiffness matrix of cross-section. The stiffness matrix \( K_t \) in Eq.(7b) includes the material nonlinearity and coupling interaction between large displacement and internal force. The element is integrated around the reinforced concrete cross-section and along the beam-column elemental axis with Gauss integration method, which makes the integral of RC nonlinear beam-column element simple and accurate (Bo et al., 2001).

3 Cumulative damage model of RC frames

Based on theory of damage mechanics, damage estimate model of structure is derived under monotonic loading in this part, which is assumed that the initial scalar is the work done under ideal non-damage state.

Figure 1 shows force-displacement skeleton curve of structure under monotonic loading, in here, \( \Delta_i \) is displacement of structure, \( K_0 \) is initial stiffness (corresponding to the line \( OA \)), then work done of structure under ideal non-damage state may be expressed as:

\[
W_p = S_{OAC} = \frac{1}{2} K_0 \Delta_i^2
\]

\[(8)\]

But actual structure is in damage stage, loading paths is non-linear. Work done may be expressed as:

\[
W_e = S_{OBC} = \int P_i d\Delta_i = \int f(\Delta_i) d\Delta_i
\]

\[(9)\]

Where \( S_{OBC} \) is areas surrounded by curve \( OB \), line \( BC \) and \( CO \); \( P \) is force; \( f(\Delta_i) \) is loading function.

Then energy dissipation may be expressed as:
\[ W_D = W_p - W_E = \frac{1}{2} K_0 \Delta_i^2 - \int_0^\Delta f(\Delta_i) d\Delta_i \]  

Based on theory of damage mechanics, damage estimate model of structure is for:

\[ D_{strui} = \frac{\frac{1}{2} K_0 \Delta_i^2 - \int_0^\Delta f(\Delta_i) d\Delta_i}{\frac{1}{2} K_0 \Delta_i^2} \]  

Where \( D_{strui} \) be known as damage estimate index of structure at \( i \)th story, the \( D_{strui} \) value is between 0 and 1.

4 Application of derived model

In order to apply suggested model, 2 specimens, the 2-story 2-span RC frame structure with L-shaped column were experimentally studied, specimen number YXKJ-1 and YXKJ-2, mainly parameters including proportion 1/3, flange thickness ratio 1/2.5, shear span ratio \( \lambda = 1.54 \), design value of concrete degree and steel bar is C30 and HPB235 respectively, test value \( f_{ck} \) of concrete is 23.96MPa. Dimensions and reinforce details of model frame is shown in figure 5. Using electric servo loading system, quasi-static test were carried out under cyclic loading, Experimental results are listed in table 1.

4.1 Comparison of analysis results with experiments

The nonlinear analysis program SAT-D for RC structures with arbitrary cross-sections is developed, based on the derived elemental model and integrated methods. Load- displacement incremental is controlled with arc-length method, nonlinear equations are solved using Newton-Simpson iteration method. Divided node and element are shown in figure 2, comparison between calculated result and test result are shown in figure 3. Calculation results are in good agreement with the experimental results from the figure. But after yield of structure, calculating displacement value are less than experimental values, Due to without considering the bond-slip between steel bars and concrete, and stiffness degradation in calculation model.

![Figure 2. Node and element mesh of structure](image-url)
4.2 Evaluation of cumulative damage

Cumulative damage index of model structure are calculated using Eq.(11), cumulative damage index of model structure are listed in the table1, figure 4 shows is destroy photo of specimen XYKJ-2 \( D_{\text{inv}} =0.940 \). Data in table 1 indicates that damage index is small in the initial crack stage, damage index is about 0.4 when crack width is 0.2mm, yield stage about 0.7, but destroy \( D_{\text{inv}} =0.95 \), near by 1.0. In short, with the load increases, increasing damage index is also increased.

![Destroy photo of specimen XYKJ-2](image1)

According to data for table1 and reference (Jinping et. al., 1999), damage index and damage degree are divided into 5 damage levels: (1) structure is in good condition when damage index \( D_{\text{inv}} \) is 0~0.2; (2) structure is in slight damage condition when damage index \( D_{\text{inv}} \) is 0.2~0.4, crack width less than 0.2mm; (3) structure is in middle damage condition when damage index \( D_{\text{inv}} \) is 0.4~0.6, crack width development from 0.2mm to steel bar yield; (4) structure is severely damaged when \( D_{\text{inv}} =0.6~0.9 \), from steel bar yield to destroy; (5) structure may be collapsed when \( D_{\text{inv}} >0.9 \).

<table>
<thead>
<tr>
<th>Number of specimen</th>
<th>initial crack</th>
<th>0.2mm crack width</th>
<th>yield</th>
<th>ultimate</th>
<th>destroy</th>
</tr>
</thead>
<tbody>
<tr>
<td>YXKZ-1</td>
<td>0.018</td>
<td>0.373</td>
<td>0.660</td>
<td>0.780</td>
<td>0.953</td>
</tr>
<tr>
<td>YXKZ-2</td>
<td>0.020</td>
<td>0.378</td>
<td>0.680</td>
<td>0.760</td>
<td>0.940</td>
</tr>
</tbody>
</table>
5 Conclusion

According to suggested nonlinear element model and damage estimating model of RC frames and experimental data in this paper, conclusions are as following:

(1) The derived element stiffness matrices are suitable for RC structures with arbitrary cross-section members.

(2) Suggested damage model under cycling load may comprehensive may reflects structural characteristics, such as energy dissipation, strength and stiffness degradation etc.

(3) Damage model is applied to analyze RC frame with L-shaped columns, the results indicated that the model can better describe the state of structural damage stage.

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References


