Plastic axisymmetric collapse of circular cylinders and cones under uniform external pressure

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Abstract

The paper presents a non-linear analysis of three circular cylinders, together with a circular cone; which all suffered plastic axisymmetric collapse under external hydrostatic pressure. The theoretical analysis used the finite element method, adopting the commercial computer package, namely ANSYS. The theoretical analysis included both geometrical and material non-linearity, where a step-by-step incremental method was used; in total 50 incremental steps were used per model. Work is also described on experimental work, which was carried out in an earlier analysis. Theoretical plots of the pressure-deflection relationships for all four models are given, together with experimental plots of pressure-strain relationships for the three circular cylinders. Comparisons between experiment and theory were good and this was found to be particularly encouraging, as the ANSYS computer analysis used the 8-node isoparametric shell element, namely Shell93; which was not axisymmetric.

Keywords: submarines, cylinders, external pressure, buckling, ANSYS

1 Introduction

Submarine pressure hulls usually consist of a combination of circular cylinders, cones and domes, as shown in figure 1. In this paper, we will limit ourselves to a study of the thin-walled circular cylinder and the circular cone, (Ross, 2001). Under external hydrostatic pressure a thin-walled circular cylinder will buckle in either a lobar manner; also known as shell instability, (Windenburg & Trilling, 1934), or due to axisymmetric deformation. In this paper, we will consider the axisymmetric mode of failure, where the vessel under external hydrostatic pressure, keeps its circular form, while imploding inwardly, as shown in figure 2.

Figure 1, A submarine pressure hull
Apart from military uses, submarines are gaining popularity in the use of commercial exploitation of the deep seas. For example, (Dickens et al, 1997) has discovered that there are about 10 000 billion tonnes of frozen methane hydrates some 3km to 11 km below the sea level. The value of this deep sea methane is some 536 times the annual GDP of the USA, or about $1.25 million per person on Earth! Ocean engineers are still not quite sure how to successfully retrieve this methane. In this paper, we will carry out a theoretical and an experimental analysis of the axisymmetric mode collapse of a submarine pressure hull; which is one of three modes of collapse.

A description of the experimental mode of collapse, followed with the theoretical analysis by ANSYS will now be given.

2 Experimental Analysis

The external pressure was increased by a hand-driven hydraulic pump and the strains noted for each value of pressure. Three of the models were circular thin-walled circular cones, namely Cones A, B & C; which collapsed axisymmetrically as shown in figure 2. The three other models were thin-walled circular cylinders, namely cylinders 4, 5 & 6; which also collapsed axisymmetrically as shown in figure 3 (Ross and Johns, 1998). The vessels were tested to destruction, under uniform external pressure, in a test tank similar to that of figure 4. The material of construction for the models was EN1A mild steel and the load-deflection relationship for this material is given in (Ross and Johns, 1998).
The circular cylinders had electrical resistance strain gauges connected to the inner surfaces of their mid-lengths and a typical Pressure-Strain distribution for cylinder 4 is shown in figure 5.
The experimental collapse pressures for the vessels are shown in table 1.

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Experiment (MPa)</th>
<th>ANSYS (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder 4</td>
<td>9.724</td>
<td>7.457</td>
</tr>
<tr>
<td>Cylinder 5</td>
<td>11.172</td>
<td>8.67</td>
</tr>
<tr>
<td>Cylinder 6</td>
<td>13.172</td>
<td>10.64</td>
</tr>
<tr>
<td>Cone A</td>
<td>5.97</td>
<td>5.68</td>
</tr>
<tr>
<td>Cone B</td>
<td>6.03</td>
<td>5.68</td>
</tr>
<tr>
<td>Cone C</td>
<td>6.21</td>
<td>5.68</td>
</tr>
</tbody>
</table>

For Cones A, B & C, their collapsed bays were of similar size and this is why their experimental collapse pressures were similar in magnitude. In the case of cylinders 4, 5 & 6, cylinder 4 was longer than cylinder 5 and cylinder 5 was longer than cylinder 6.

3 Theoretical Analysis

The theoretical analysis was based on the finite element method, using the commercial computer package, namely ANSYS. The method used was a step-by-step incremental method; which allowed for both material and geometrical non-linearity. The method will now be described with the aid of figure 6.

In figure 6, a small incremental pressure load was applied, and the resulting displacements and stresses calculated; in this case the geometrical stiffness matrix \([K_G]\) was a null matrix. Having calculated the stresses due to the small incremental pressure of Step 1, it was now possible to calculate the geometrical stiffness matrix and the new stiffness matrix \([K]\) allowing for a change in geometry and any changes in the material properties of any of the elements. Step 2 was now implemented and the resulting incremental displacements and stresses added to those of Step 1; allowing for material and geometrical non-linearity. If the stress in any element exceeded the yield
stress, the Young’s Modulus for that element was set to 1/100 th the Young’s Modulus at the start. A total of about 50 steps were made, until a theoretical catastrophic collapse took place.

The mesh for a typical circular cylinder is shown in figure 7. The theoretical plots for the Pressure Versus Displacements for cylinders 4, 5 & 6 are shown in figures 8, 9 & 10; the cones’ plots were similar.

Figure 7, A mesh for a typical circular cylinder

Figure 8, ANSYS pressure-deflection plot for cylinder 4

Figure 9, ANSYS pressure-deflection plot for cylinder 5
Comparison with experiment and theory are shown in table 1.

4 Conclusions

The paper has shown that the commercial computer package, namely ANSYS, is suitable for analysing the plastic axisymmetric collapse for thin-walled circular cylinders and cones under external hydrostatic pressure. The authors believe that the method is also applicable to full-scale submarines, constructed from ductile metals.

References


