An integrated finite strip solution for dynamic analysis of long span bridges

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Abstract

This paper introduces a total integrated analytical solution for multi-span, continuous slab-on-girder and box girder bridges by modelling the bridge deck and the piers together, using finite strip method (FSM). FSM has been well accredited for its efficiency in the structural analysis of bridges, reducing the time required for data input and analysis without affecting the degree of accuracy. By using a continuously differentiable smooth series in the longitudinal direction, a complex 3D problem is reduced to a 2D problem using the FSM. However, difficulties are encountered when components of different orientation, such as the piers, are being added to the formulation. Thus, the analytical model developed using the conventional FSM is limited to the super-structures without proper consideration of the interactions between the bridge deck (super-structure) and piers (sub-structure). In this regard, the cantilever type of pier strip element is formulated by the authors, based on the spline finite strip concept, which is compatible with the well developed spline finite strip bridge deck. In addition, by combining the piers and the bridge deck together in a single finite strip formulation, with some appropriate connecting boundary conditions, the time required for both static and dynamic analysis can be significantly reduced. In this paper, the development and verification of the vertical cantilever strip is introduced and the overall integrated method of analysis is presented with the aid of numerical examples. In addition, the efficiency of the proposed approach in seismic analysis using the Pseudo Excitation Method (PEM) is also demonstrated as an extension of its application.

Keywords: Finite Strip Method, Pseudo Excitation Method, Dynamic Analysis, Box-girder Bridge.

1 Introduction

Quasi-static analysis approach in bridges design has been adopted for over a century. However, with the successful application of stiff and light-weight composite materials in recent years, the clear span length of bridges nowadays are increasing at a faster pace than ever. Consequently, the critical design criteria shifted from the static ultimate and serviceability considerations to the dynamic driven failure mechanisms. Nevertheless, the formulation of a FEM model and the setting up of boundary conditions of a three dimensional (3D) bridge for dynamic analysis is very complicated and time consuming, and this makes the dynamic analysis of long span bridges extremely difficult in real design process. In this regards, FSM seems to have provided an ideal solution by reducing the input-output time requirement and the computational loading demand. FSM has been well-recognized as one of the most efficient tools for the analysis of bridge super-structures due to its semi-analytical nature as well as its pre-set boundary conditions (Cheung, Li and Chidiac, 1996). Wang and Zhang
(2004) summarized the advantages and constraints of FSMs in a recent review. The FSM is an ideal approach in analyzing the dynamic properties of bridge deck structures, especially when dealing with slab-on-girder and box girder structures that have pre-set boundary conditions. In spite of the large number of publications on the use of FSM in superstructure analysis, the application of existing FSM in full-bridge analysis is limited as it is not possible to insert extra component at an intermediate point within a strip from the finite strip point of view. In this paper, a general integrated framework is developed to solve the static and dynamic structural problems in an efficient and vigorous manner, under the FS environment.

2 Pier Strips in finite strip environment

The formulation of the conventional strip elements is limited to some common boundary condition for bridge desk only. To provide a total solution for bridge analysis in the finite strip formulation, with the consideration of the piers, a special type of strip element is derived from the principle of finite strip concept. To maintain compatibility and uniqueness throughout the whole structure, the more general B3 spline finite strip function in the longitudinal direction is adopted as the basis in the development of the column strip’s displacement field. In the transverse direction, a cubic polynomial is applied to represent the variation of the vertical displacement, whereas linear interpolation is adopted for the in-plane displacements. The B3 spline function is a piecewise cubic polynomial with continuity over the entire interval up to the second derivative. For a conventional B3 spline finite strip, in order to interpolate an arbitrary function \( f(y) \) by the spline functions, \( f(y) \) is divided into several sections, known as knots. For an equally spaced spline function \( \Phi_m(y) \) with the centre at \( y=y_m \), the B3 spline function is defined as (Prenter, 1975):

\[
\Phi_m(y) = \frac{1}{6h} \left\{ \begin{array}{ll}
(y-y_m)^3, & y_m \leq y \leq y_{m+1} \\
h^3 + 3h^2(y-y_m) + 3h(y-y_m)^2 - 3(y-y_m)^3, & y_{m+1} \leq y \leq y_m \\
(y-y_{m+2})^3, & y_{m+2} \leq y \leq y_{m+1} \\
0, & \text{otherwise}
\end{array} \right.
\]

in which \( h \) is the width of these equal sections. The solution of each section is in connection with the four spline functions, which are the functions centred at the two ends of the section and the two knots next to those ends, respectively. Thus, two additional knots are needed to complete the interpolation of the whole function. For a strip divided into \( r \) sections, \( r+3 \) B3 spline functions are needed. To develop the piers strip, it is necessary to achieve a continuous shape function satisfying the bending behaviour and the boundary conditions for a cantilever-behaved pier. Consider a vertical cantilever strip, fixed at one end while leaving the other end free, as shown in Fig. 1. The global z-direction of the column strip is similar to the local v-direction in the conventional spline finite strip, controlled by the in-plane stiffness in the corresponding direction. Similarly, the global v-direction of the column strip is similar to the local z-direction of the conventional spline finite strip. According to the above information, the displacement function for the column strip can be expressed as:

\[
U = \sum_{m=0}^{r+3} \left\{ (1-X)u_m + Xu_n \right\} \Phi_m(y)
\]

\[
V = \sum_{m=0}^{r+3} \left\{ (1-X^2+2X^3)v_m + x(1-2X+X^3) \right\} \Phi_m(z) + x(3X^2-2X^3)v_m + x(X^2-X) \Phi_m(z)
\]

\[
W = \sum_{m=0}^{r+3} \left\{ (1-X)w_m + Xw_n \right\} \Phi_m(z)
\]

With the assumed displacement functions and the preset boundary conditions, the shape function, \( N \), can be developed using traditional finite element concepts. It should be noted that all strip elements in finite strip method must come with the preset boundary conditions. Therefore, the strip developed for the piers must satisfy the fixed-free boundary condition as for a vertical cantilever. The stiffness matrix and the mass matrix can be calculated using the following equations:
\[ [M_0]_{mn} = \int \rho h[N]^T[N]_m dA; \quad [K_0]_{mn} = \int [B]^T[D][B]_m dA \]

in which \( \rho \) is the density; \([D]\) and \([B]\) are the elastic matrix and the strain matrix respectively.

\[ \text{Figure 1, Vertical cantilever strip element – Column Strip} \]

3 Transition Section: Interactions between Deck and Pier

A strip is ideally defined by the preset boundary conditions at the two ends and some intermediate knots only. The physical properties in the longitudinal direction of the strip are modelled by a continuously smooth functions and the concept of element or node is unavailable. Thus, without the nodal concept, the piers and the superstructure cannot be connected under the conventional finite strip formulation, and this is becoming the major obstacle for the application of FSM in real practice. Although knots are available within the SFS, they are there for spline interpolation only. Unlike the nodal definition in FEM, the properties of each knot in spline finite strip method (SFSM) is a function of the properties of surrounding knots and a minimum of 2 extra knots is required to define the properties of a particular knot. Although it is impossible to connect extra element to a particular knot, it is made possible by introducing a tiny rigid element to connect two knots in each strip for force and displacement transfer. Fig. 2 demonstrated a typical cross-shaped transition section, defined that \( h \) is much smaller than \( H \). By connecting two knots on each side, the displacement of the internal knots within the rigid transition section can be defined. Thus, compatibility for the displacements of the deck and pier is achieved, and the integrated system of the spline finite strip model can be constructed. Depending on the structure complexity, one may choose different values of \( h \) to meet the required accuracy. The numerical study shows that, in most of the cases, \( h/H=0.001 \) is good enough to achieve an acceptable tolerance of error (less than 0.5%), for engineering analysis.

\[ \text{Figure 2, Transition section between Deck and Pier} \]

4 Numerical Example

4.1 Slab-on-girder Bridge – Accuracy and efficiency

A concrete slab bridge model, as shown in Fig. 3, is presented here. The modulus of elasticity \( E = 3.0 \times 10^4 \text{ MPa} \), the Poisson’s ratio is 0.2 and the material density is 2500 kg/m³. Both SFSM and
FEM are adopted to model the static and dynamic behavior of the structure, and the results from different methods are compared. Three numerical methods were adopted. For the integrated SFS model, a full bridge model is constructed, using B_s spline finite strip for the deck, vertical-cantilever strip for the pier and transition elements for the bearing, as illustrated in the previous sections. The deck is divided into four equal strips, while the pier is divided into two cantilever strips. Each deck strip is composed of 32 sections as well as two additional transition sections. Each pier strip is composed of 4 sections, as well as one transition section. For the coarsely meshed FE model, the mass and stiffness of the bridges is constructed with five degrees-of-freedom shell elements throughout the structure. The shell element is derived from a combination of an in-plane element with translation in the x and y directions, and a bending element with translation in the z direction, plus bending above x and y directions. The deck is meshed with $32 \times 4$ elements, and the pier is meshed with $4 \times 2$ elements. For the finely meshed FE model, which is an indicator for a more accurate solution, standard shell elements are employed to construct the model, with six degrees-of-freedom at each node. For the static analysis, the ability for the transition element to transfer loading between the deck strip and the column strip (CS) is assessed. Four load cases, with constant 1000kN point forces acting at different structure components and in different directions, were assigned to the models as shown in Fig. 3. Typical drifts along nodal lines are summarized in table 1. It is suggested that the displacement calculated from the integrated approach agrees well with FEM results in all loading conditions, which indicates that the proposed approach can successfully model a full bridge structure in the finite strip environment, taking consideration of the pier-bearing-deck interaction.

![3D bridge model and SFSM model configuration.](image)

It is suggested that the displacement calculated from the integrated approach agrees well with the FEM results in all loading conditions, which indicates that the proposed approach can successfully model a full bridge structure in the finite strip environment, taking consideration of the pier-bearing-deck interactions. Besides, it can be observed from the static analysis result that the displacement determined from the integrated method agrees better with the finely meshed FE model. It is suggested that, even with similar node definitions as the coarsely mesh FE model, the proposed integrated approach generates a more accurate result than the FE method.

| Table 1: Load Case (a) | deck: nodal line 3 | translation: v (m) | FEM | Integrated | SFSM | FEM | Coarse mesh | 0.00006522 | 0.00005738 | 0.00006300 | -0.00079888 | -0.00072660 | -0.00075700 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| y (m) | | | | Integrated | SFSM | Coarse mesh | Refined Mesh | Integrated | SFSM | Coarse mesh | Refined Mesh | Integrated | SFSM | Coarse mesh | Refined Mesh |
| 0 (left) | | | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | | | |
| 16 | | | | | | | | | | | | | | |
| 24 | | | | | | | | | | | | | | |
| 32 (right) | | | | | | | | | | | | | | |

pier: nodal line 7

| y (m) | | | | | | | | | | | | | | |
| 0 (top) | | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | |
Finite strip method has been widely recognized for its efficiency in numerical analysis. The above computations were executed on a personal computer with Intel Core2 Duo CPU (1.66 GHz) and 3GB physical memory. The computer times required by different methods are compared in table 2. It is obvious that the SFSM is more efficient and saves about 10% computer time. Considering it is only the static analysis, the proposed SFSM will save more time for complicated dynamic analysis. Moreover, in order to achieve matching preconditions for analysis, similar meshes are chosen in the SFSM and the FEM models. However, due to its semi-analytical property in the longitudinal direction, the number of sections for each strip in the SFSM model could be largely reduced without losing accuracy. Then the time for computation could be reduced, and greater efficiency could be achieved.

<table>
<thead>
<tr>
<th>Method</th>
<th>SFSM</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer time</td>
<td>4.717s</td>
<td>5.296s</td>
</tr>
</tbody>
</table>

### 4.2 Box-girder Bridge – Dynamic analysis

A typical three-span concrete box girder bridge model illustrated in Fig. 4 is constructed by the proposed approach for further analysis. The deck and piers are all modeled by four degrees-of-freedom B₃ spline finite strips and the CSs respectively. One transition section is adopted at each connection between the deck and the pier. For all the components, the modulus of elasticity is \( E = 3.0 \times 10^4 \text{ MPa} \), Poisson’s ratio is 0.2, and the material density is 2500 kg/m³.

![Figure 4, 3D bridge model; transverse cross-section & numerical model for the three-span box girder bridge.](image)

In order to optimize the advantages of the integrated finite strip solution, in terms of precision and efficiency, the authors adopted the Pseudo Excitation Method (PEM) for the dynamic analysis to obtain the response probability spectrum density (PSD) of the structure. The recently developed PEM is an accurate and highly efficient dynamic analysis alternative for long-span structures (Zhang, Li, Lin and Williams, 2009). With the characteristic property matrices well defined in the FS environment, the conventional characteristic equation of motion can be constructed, and the dynamic analysis procedure, using the same approach as for FEM, can be applied. When structure is subjected to a uniform earthquake excitation, the dynamic motion equation can be modified as:

\[
[M]\ddot{\mathbf{u}} + [C]\dot{\mathbf{u}} + [K]\mathbf{u} = -[M]\mathbf{r} \sqrt{S_{u_g}(\omega)} e^{i\omega t} \tag{4}
\]

in which \( S_{u_g}(\omega) \) is the auto-PSD of the ground acceleration. By applying the conventional response analysis method for multi-degree-of-freedom systems, the pseudo displacement parameters of SFSM can be computed by the equation as:

\[
\{\mathbf{a}(\omega,t)\} = \sum_{j=1}^{n} \gamma_j H_j \sqrt{S_{u_g}(\omega)} e^{i\omega t}; \quad H_j = \frac{1}{\omega_j^2 - \omega + 2i\zeta_j \omega}\quad \gamma_j = -[\phi_j]^T[M]\mathbf{r} \tag{5}
\]

Here, \( \omega_j \) is the \( j \)th angular frequency associated with matrices M and K; \( [\phi_j] \) is the corresponding normalized mode; \( \zeta_j \) is the \( j \)th damping ratio. By substituting the pseudo displacement parameters
into the displacement functions, the pseudo displacement responses are obtained. Hence, the PSD matrix of displacements could be computed from:

\[ S_\omega (\omega) = \{ \tilde{U}(\omega,t) \}^T \{ \tilde{U}(\omega,t) \} \]  

(6)

In this analysis, multiple support excitations with wave passage effect are considered. An artificial seismic wave, illustrated in Fig. 5, is selected with the acceleration record acting in the transverse direction. The velocity of the seismic wave along the longitudinal direction is assumed to be 100m/s.

The PSD curves of the displacement responses of selected points on deck and pier (Fig. 4) are illustrated in Fig. 5. Since PSD indicates the magnitude of the energy as a function of frequency, it can be observed that these curves capture the characteristics of the structure response under the described seismic wave, which include the peaks of the first few natural frequencies and the drop of energy after about 10 Hz. The overall PSD curve of pier is lower than the curve of deck; however, dynamic response of the pier can be quite critical at some frequencies. Moreover, for normal bridges, the piers are often stiffer than the deck, and the piers are more likely to be damaged during an earthquake. That is the reason why the pier must be considered in dynamic analysis and why the integrated finite strip solution is developed.

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References